**10 .... 0(2)=2^n**

**n**

**1... .... 1(2)= 10.....0(2) -1(2)= 2^n -1**

**n n**

**Integer numbers – codes and operations in complementary code**

**Example 1**

**n=8 bits**

**X= 56**

**Y= 99**

**X=56**

**56=32+16+8=2^5+2^4+2^3 =111000(2)**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **positions** | **S 7 6 5 4 3 2 1 0** | | | | | | | |
| **[56] dir = [56]inv= [56]compl =** | **0** | **0** | **1** | **1** | **1** | **0** | **0** | **0** |
| **[-56]dir =** | **1** | **0** | **1** | **1** | **1** | **0** | **0** | **0** |
| **[-56]inv =** | **1** | **1** | **0** | **0** | **0** | **1** | **1** | **1** |
| **[-56]compl =** | **1** | **1** | **0** | **0** | **1** | **0** | **0** | **0** |

**Y= 99 = 64 + 32 + 2 + 1 = 2^6 + 2^5 + 2^1 + 2^0 = 1100011(2)**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **positions** | **S 6 5 4 3 2 1 0** | | | | | | | |
| **[99] dir = [99]inv= [99]compl=** | **0** | **1** | **1** | **0** | **0** | **0** | **1** | **1** |
| **[-99]dir =** | **1** | **1** | **1** | **0** | **0** | **0** | **1** | **1** |
| **[-99]inv =** | **1** | **0** | **0** | **1** | **1** | **1** | **0** | **0** |
| **[- 99]compl =** | **1** | **0** | **0** | **1** | **1** | **1** | **0** | **1** |

**[56+99]compl = [56]compl  [99]compl**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S** | | | | | | | |  | **Overflow,**  **The operands are positive, the result is negative** |
| **[56]compl =** |  | **0** | **0** | **1** | **1** | **1** | **0** | **0** | **0** | **** |
| **[99]compl =** |  | **0** | **1** | **1** | **0** | **0** | **0** | **1** | **1** |  |
|  |  | **1** | **0** | **0** | **1** | **1** | **0** | **1** | **1** |  |

**[99-56]compl = [99]compl  [-56]compl**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S** | | | | | | | |  | **Correct result**  **2^0+2^1+2^3+2^5=1+2+8+32=43** |
| **[99]compl =** |  | **0** | **1** | **1** | **0** | **0** | **0** | **1** | **1** | **** |
| **[-56]compl =** |  | **1** | **1** | **0** | **0** | **1** | **0** | **0** | **0** |  |
| **[43]compl** | **~~1~~** | **0** | **0** | **1** | **0** | **1** | **0** | **1** | **1** |  |

**[56-99]compl = [56]compl  [-99]compl**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S** | | | | | | | |  | **Corr ect result** |
| **[56]compl** |  | **0** | **0** | **1** | **1** | **1** | **0** | **0** | **0** | **** |
| **[-99]compl =** |  | **1** | **0** | **0** | **1** | **1** | **1** | **0** | **1** |  |
| **[-43]compl** |  | **1** | **1** | **0** | **1** | **0** | **1** | **0** | **1** |  |
|  |  | **0** | **0** | **1** | **0** | **1** | **0** | **1** | **1** |  | **complement** |

**Subunitary numbers – codes and operations in complementary code**

**Example 2**

**n=8 bits**

**X= 0,45**

**Y= 0,77**

**X=0,45==0,0111001(2)**

**0,45\*2=0,9**

**0,9\*2=1,8**

**0,8\*2=1,6**

**0,6\*2=1,2**

**0,2\*2=0,4**

**0,4\*2=0,8**

**0,8\*2=1,6**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **positions** | **S 7, 6 5 4 3 2 1 0** | | | | | | | |
| **[0,45] dir = [0,45]inv= [0,45]compl =** | **0** | **0** | **1** | **1** | **1** | **0** | **0** | **1** |
| **[-0,45]dir =** | **1** | **0** | **1** | **1** | **1** | **0** | **0** | **1** |
| **[-0,45]inv =** | **1** | **1** | **0** | **0** | **0** | **1** | **1** | **0** |
| **[-0,45]compl =** | **1** | **1** | **0** | **0** | **0** | **1** | **1** | **1** |

**Y=0,77= 0,1100010(2)**

**0,77\*2=1,54**

**0,54\*2= 1,08**

**0,08\*2=0,16**

**0,16\*2=0,32**

**0,32\*2=0,64**

**0,64\*2=1,28**

**0,28\*2=0,56**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **positions** | **S 7, 6 5 4 3 2 1 0** | | | | | | | |
| **[0,77] dir = [0,77]inv= [0,77]compl=** | **0** | **1** | **1** | **0** | **0** | **0** | **1** | **0** |
| **[-0,77]dir =** | **1** | **1** | **1** | **0** | **0** | **0** | **1** | **0** |
| **[-0,77]inv =** | **1** | **0** | **0** | **1** | **1** | **1** | **0** | **1** |
| **[-0,77]compl =** | **1** | **0** | **0** | **1** | **1** | **1** | **1** | **0** |

**[0,77+0,45]compl = [0,77]compl  [0, 45]compl=**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S ,** | | | | | | | |  |  |
| **[0,77]compl =** |  | **0** | **1** | **1** | **0** | **0** | **0** | **1** | **0** | **** | **Overflow => the operands are positive and the result is negative** |
| **[0,45]compl =** |  | **0** | **0** | **1** | **1** | **1** | **0** | **0** | **1** |  |
|  |  | **1** | **0** | **0** | **1** | **1** | **0** | **1** | **1** |  |

**[0,77-0,45]compl = [0,77]compl  [-0,45]compl**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S ,** | | | | | | | |  |  |
| **[0,77]compl** |  | **0** | **1** | **1** | **0** | **0** | **0** | **1** | **0** | **** | **Correct result**  **2^(-2) + 2^(-4) + 2 ^(-7)**  **=0,25+0,0625+0,0078125**  **=0,3203125** |
| **[-0,45]compl** |  | **1** | **1** | **0** | **0** | **0** | **1** | **1** | **1** |  |
| **[0,32031]compl** | **~~1~~** | **0** | **0** | **1** | **0** | **1** | **0** | **0** | **1** |  |

**[0,45-0,77]compl = [0,45]compl  [-0,77]compl**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S ,** | | | | | | | |  |  |
| **[0,45]compl** |  | **0** | **0** | **1** | **1** | **1** | **0** | **0** | **1** | **** | **Correct result**  **-(2^(-2) + 2^(-4) + 2 ^(-7)) =**  **~=-0,32031** |
| **[-0,77]compl** |  | **1** | **0** | **0** | **1** | **1** | **1** | **1** | **0** |  |
| **[-0,32031]compl** |  | **1** | **1** | **0** | **1** | **0** | **1** | **1** | **1** |  |
|  |  | **0** | **0** | **1** | **0** | **1** | **0** | **0** | **1** |  | **complement** |

**Example 3: Represent in fixed-point notation, on 32 bits, I=14 bits, the number 3456,78**

**1+I+F=32 bits**

**3456= 2048 + 1024 + 256 + 128 = 2^11 + 2^10 + 2^8 + 2^7**

**=> 3456 = 110110000000(2)**

**0,78= 0,6172702(8) = 0,110 001 111 010 111000 010 (2)**

**0,78\*8=6,24**

**0,24\*8=1,92**

**0,92\*8=7,36**

**0,36\*8=2,88**

**0,88\*8=7,04**

**0,04\*8=0,32**

**0,32\*8=2,56**

**3456,78= 110110000000, 110 001 111 010 111000 010 (2)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| S | |  | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|  | | I=14 bits -> , <- F=17 bits | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 0 | | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | | | | | B | | | | 0 | | | | 1 | | | | 8 | | | | F | | | | 5 | | | | C | | | |

**Example 4: Represent in floating point notation, single precision (SP), the number: 3456,78**

**Mantissa<1**

**3456,78= 110110000000, 110 001 111 010 111000 010 (2)**

**3456,78= 0, 1101 1000 0000 1100 0111 101 \* 2 ^ (12)**

**mantissa**

**e= 12,**

**c = e + 127 = 12 + 127 = 139 = 2^7 + 2^3 + 2^1 + 2^0 = 10001011 (2)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | |  | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | | c=e+127 (8bits) -> | <- mantissa (23 bits) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 1 | | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 4 | | | | | 5 | | | | E | | | | C | | | | 0 | | | | 6 | | | | 3 | | | | D | | | |

**Mantissa>1**

**3456,78= 110110000000, 110 001 111 010 111000 010 (2)**

**3456,78= 1,101 1000 0000 1100 0111 1010 \* 2 ^ (11)**

**Mantissa**

**e= 11**

**c=11+127=138= 2^7 + 2^3 + 2^1 = 10001010 (2)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | |  | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | | c=e+127 (8bits) -> | <- mantissa (23 bits) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 1 | | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| 4 | | | | | 5 | | | | 5 | | | | 8 | | | | 0 | | | | C | | | | 7 | | | | A | | | |

**Example 5: Find the real number X having C504A800 its fixed-point representation on 32 bits with I=17 bits.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | I=17 bits -> , <- F=14 bits | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | | | | 5 | | | | 0 | | | | 4 | | | | A | | | | 8 | | | | 0 | | | | 0 | | | |

X = - 10001010000010010, 101 (2) = -(2^16 + 2^12 + 2^10 + 2^4 + 2^2 +2^(-1) + 2^(-3))=

= -(65536 + 4096 + 1024 + 16 + 4 + 0,5 + 0,125) = -70676, 625

**Example 6:** **Find the real number X having C504A800 as its floating-point representation, SP, m>1.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | |  | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | | c=e+127 (8bits) -> | <- mantissa (23 bits) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | | | | | 5 | | | | 0 | | | | 4 | | | | A | | | | 8 | | | | 0 | | | | 0 | | | |

c=10001010(2) = 2 ^7+2 ^3+2 ^1 = 138

c=138=e+127, e=11

X=- 1,00001001010100(2) \*2^11=-100001001010,100(2)=-(2 ^11+2 ^6+2 ^3+2 ^1+2 ^(-1))=

-(2048+64+8+2+0.5)=-2122,5